Cent'anni di Relatività Generale: dalla geniale proposta di Einstein alla recente scoperta delle onde gravitazionali

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Reggio Emilia

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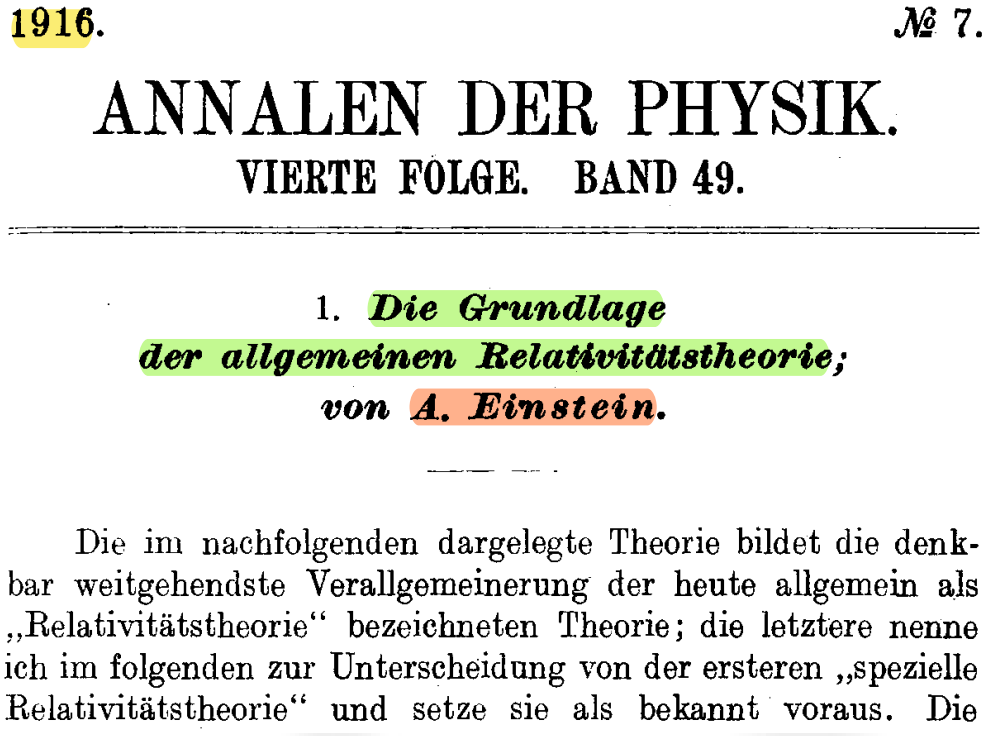
1915-1916

- Einstein's equations: late 1915
- Foundations of General Relativity: spring 2016



1916. № 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage der allgemeinen Relativitätstheorie;*
von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie" bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie" und setze sie als bekannt voraus. Die

11 May 1916

Karl Schwarzschild dies

- Schwarzschild found the first exact solution to Einstein's equations of GR, in late 1915 while at the Russian front. Paper published in 1916
- Spherical mass (e.g. star or planet)
- Mass density high enough $\rightarrow$ Schwarzschild black hole

Einstein's equations of General Relativity

$$\underbrace{G_{\mu\nu}}_{\text{Geometry}} = \underbrace{\frac{8\pi G}{c^4} T_{\mu\nu}}_{\text{Particle Physics}}$$

- Geometry: properties of space
- Particle physics: matter acts as source

Einstein's equations of General Relativity

$$\underbrace{G_{\mu\nu}}_{\text{Geometry}} + \underbrace{\frac{8\pi G}{c^4}\Lambda g_{\mu\nu}}_{\text{Cosmological Constant}} = \underbrace{\frac{8\pi G}{c^4}T_{\mu\nu}}_{\text{Particle Physics}}$$

- Geometry: properties of space
- Particle physics: matter acts as source
- Cosmological constant, "the biggest blunder of my life" –A. Einstein

What is General Relativity?

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- the interplay between THEORY (prediction) and EXPERIMENTS (test of prediction and discovery of new physics) has led to outstanding results

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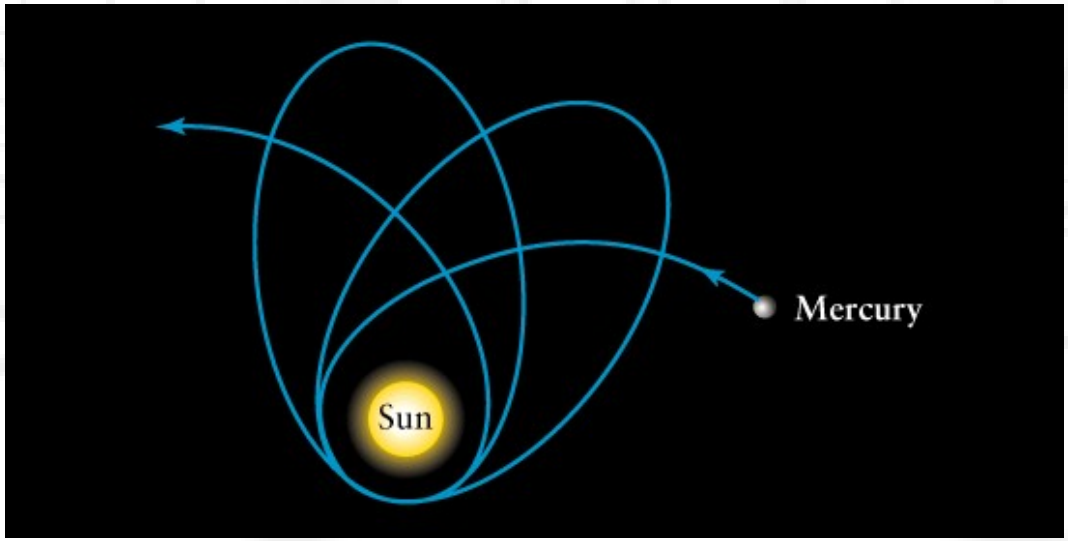
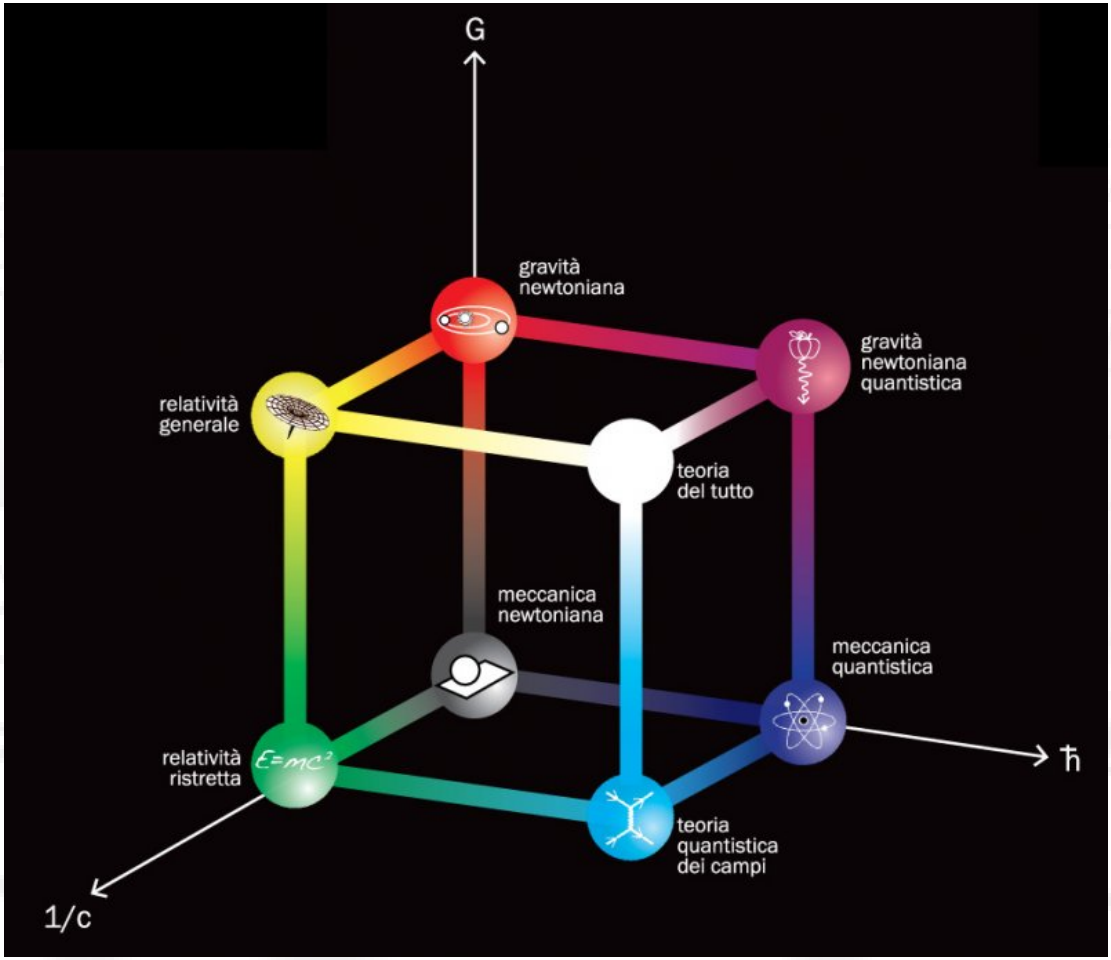
- the interplay between THEORY (prediction) and EXPERIMENTS (test of prediction and discovery of new physics) has led to outstanding results
- Examples: Mercury's anomalous orbit very accurately described by GR



...and recently, Gravitational Waves (GW) detected

What is General Relativity?

Technically, General Relativity is a Geometric, Field Theoretical description of:



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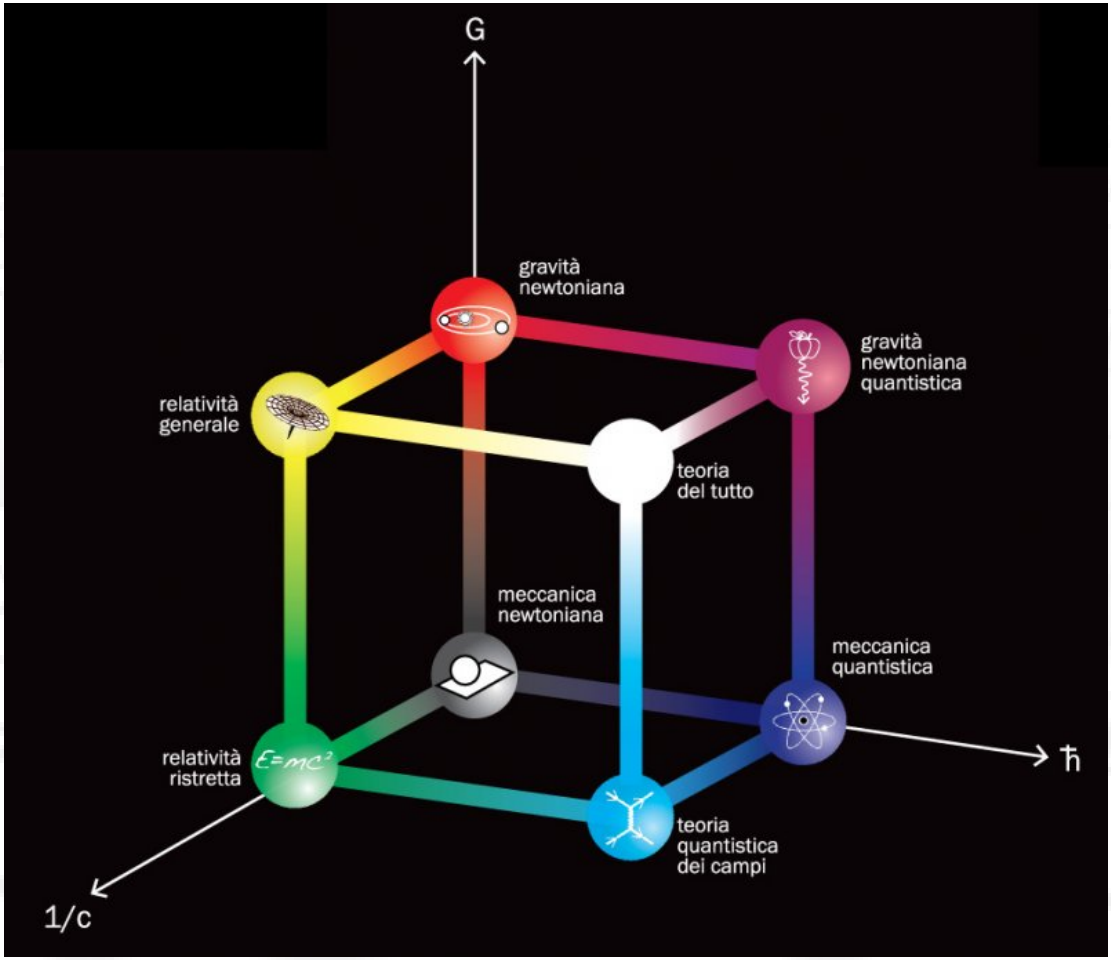

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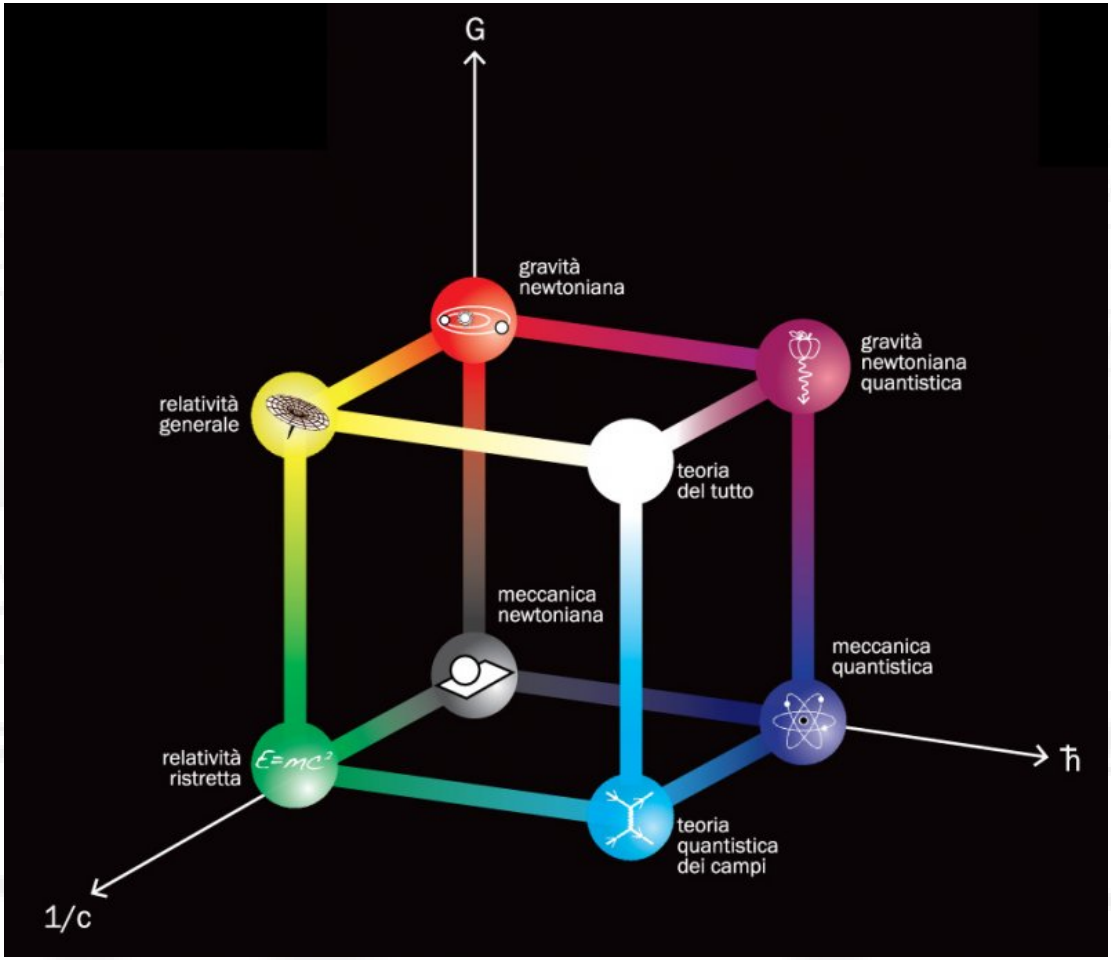
- Gravitational interactions between relativistic bodies
- Large scale structure of Universe



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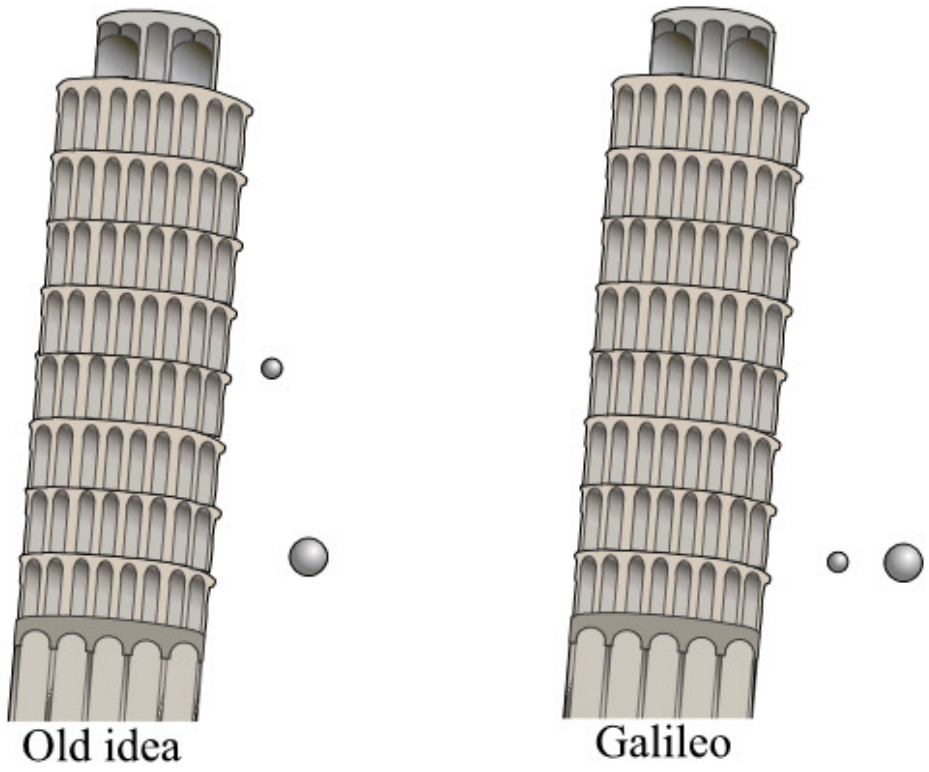
The (weak) Equivalence Principle:

What is General Relativity?

The (weak) Equivalence Principle:

- Galileo observed that the free falling motion of bodies is independent of the mass



The Universality of Free Fall

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The (weak) Equivalence Principle:

- 3rd Newton's law

$$\underbrace{m_{in}}_{\text{Inertial Mass}} \cdot \underbrace{a}_{\text{Acceleration}} = \underbrace{m_g}_{\text{Gravitational Mass}} \cdot \underbrace{g}_{\text{Gravitational Field}}$$

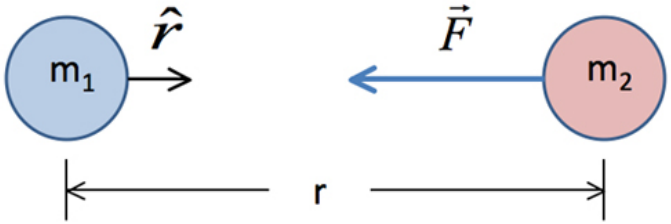
Universality of free fall:

$$a = g \quad \& \quad m_{in} = m_g$$

Einstein's GR explains why

What is General Relativity?

Newton's law of universal gravitation:



$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

satisfies the universality of free fall

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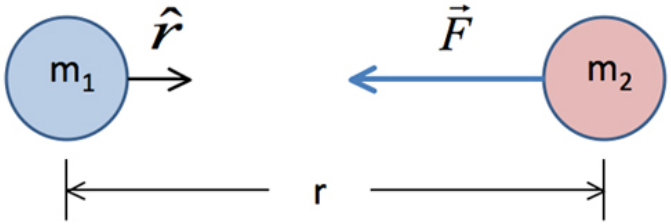

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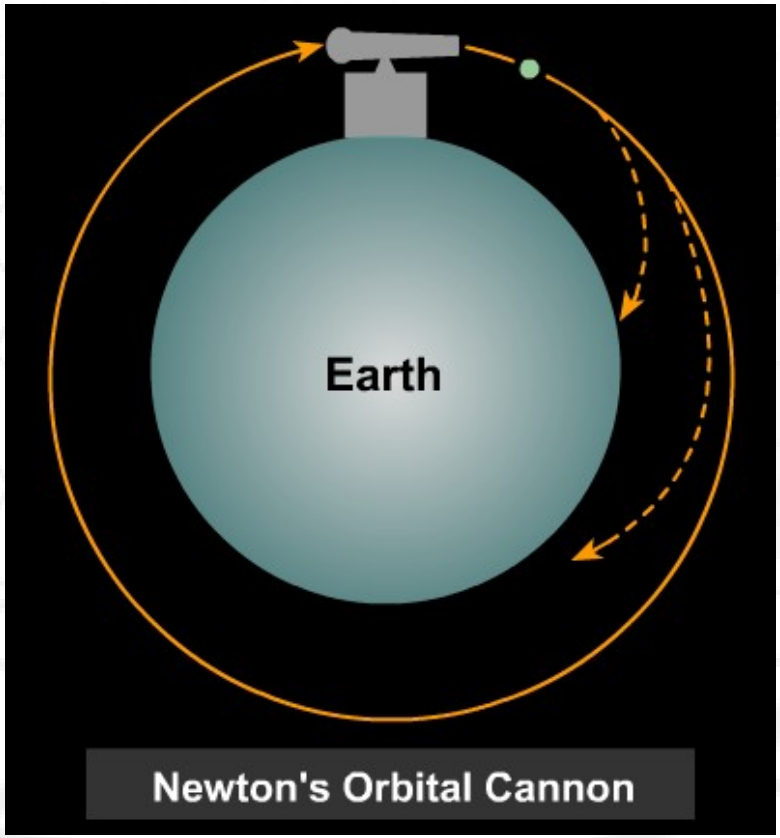
- It only depends on the source, m_1, on the Newton's constant, G and on the distance, r
- Bound orbits are closed (Bertrand's theorem)

What is General Relativity?

- Small velocity: ball falls close to the cannon



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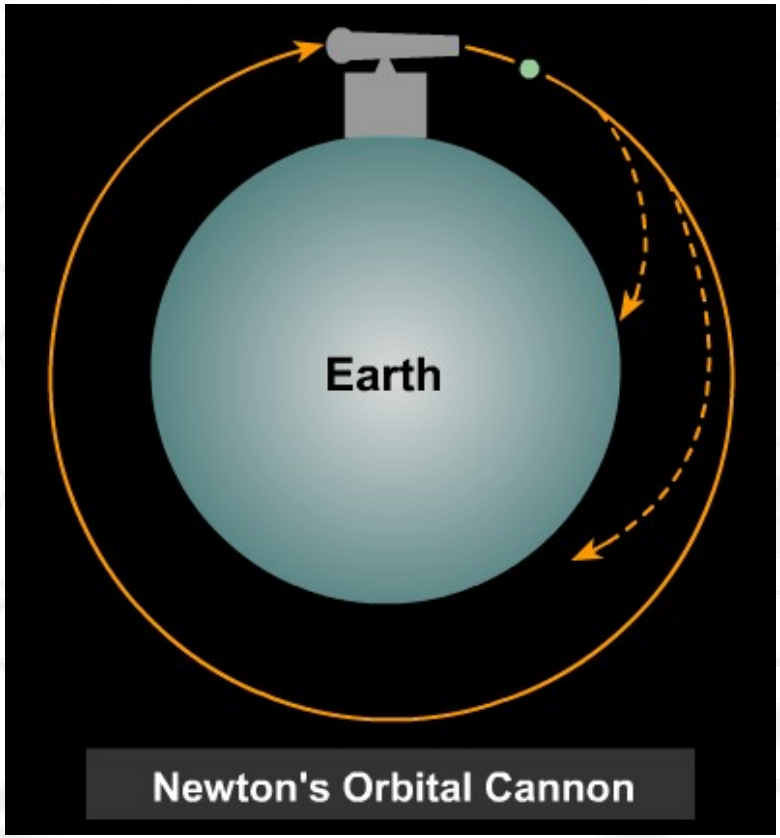

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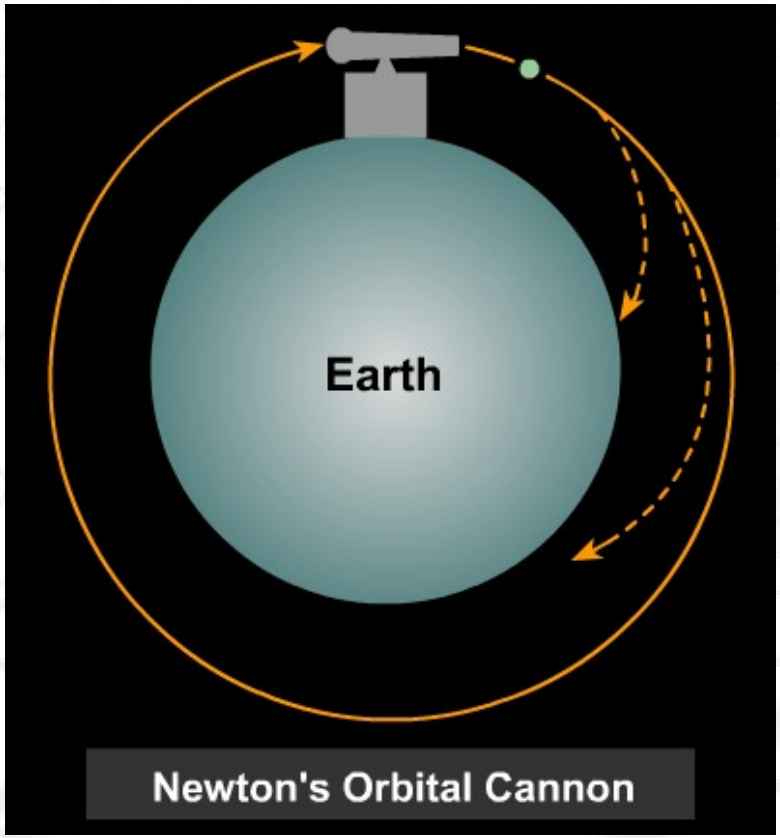

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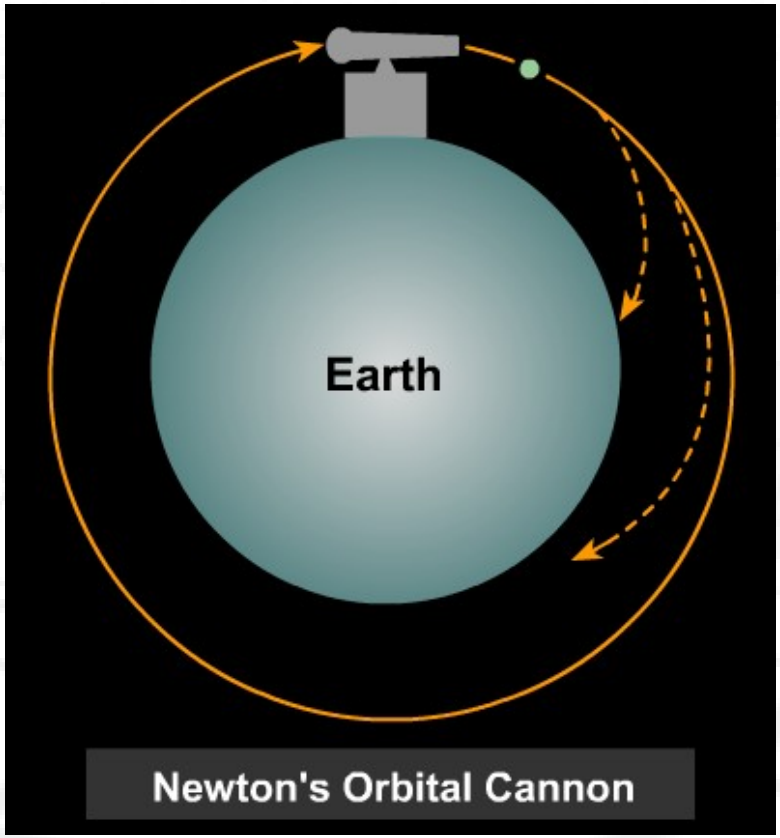
- Newton's theory unifies terrestrial gravity with celestial gravity



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- Locality: Physical signals travel at finite velocity!
  $\Rightarrow$ effects happen with time-delay after causes

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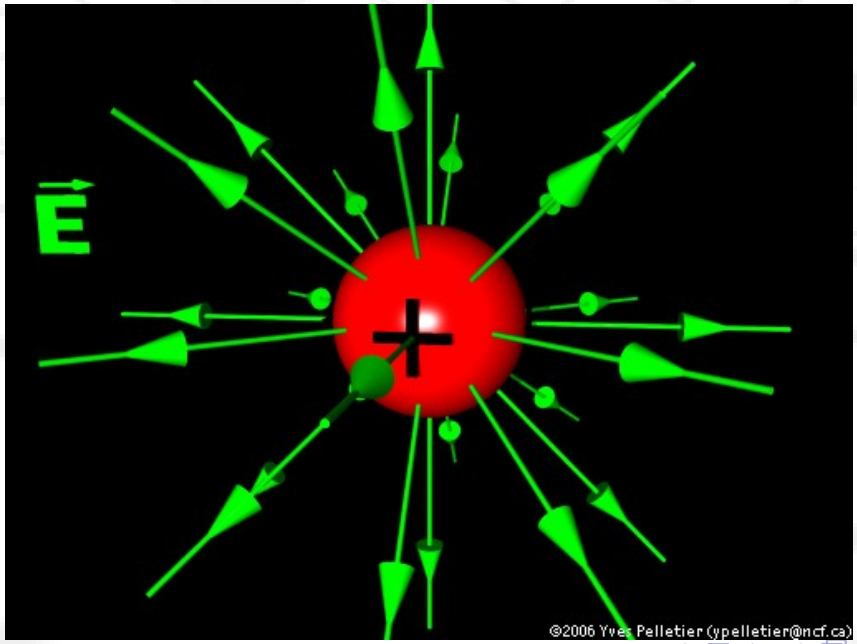
$$\vec{F} = kq'q\frac{\hat{r}}{r^2} = q'\vec{E}$$

$\vec{E}(\vec{r})$ = vector field generated by a constant charge q that sits in the origin, and exists independently of the presence of charge q'.
In each point $\vec{r}$ of space there is a vector $\vec{E}(\vec{r})$



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- Maxwell: the information is encoded in the (electro-magnetic) field, which satisfy Maxwell's equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
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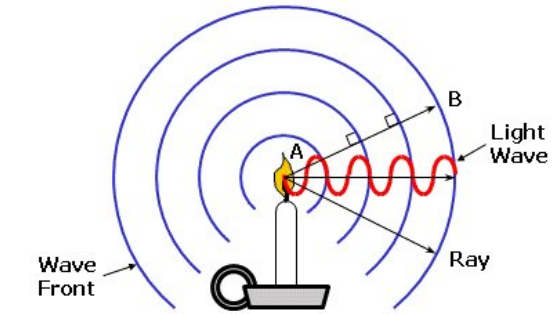
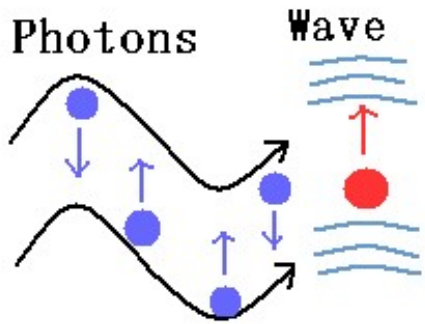
- Maxwell's equations have wave solutions: light travels at finite velocity, the speed of light
- Maxwell's equations are consistent Einstein's theory of special relativity: $\vec{E}$ and $\vec{B}$ are different manifestations of the same phenomenon

What is General Relativity?

- The information propagates in waves that travel at the speed of light



Wave Fronts Propagating From a Point Source



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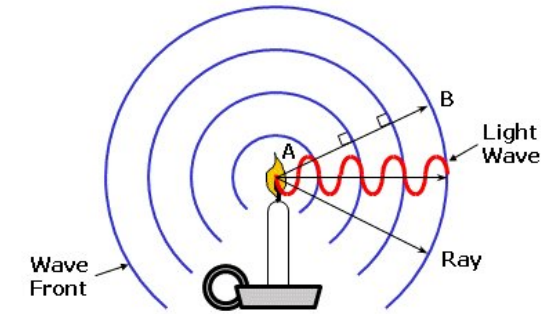
- The information propagates in waves that travel at the speed of light
- Electromagnetic waves are made of tiny particles called "photons" that are ripples of the electromagnetic field $\Rightarrow$ a photon is the same anywhere (universality)
- Photon is the "messenger" of the electromagnetic interaction
- Planck-Einstein: It carries a "quantum" of energy $E_\nu = \hbar\omega$, $\hbar$= Planck's constant



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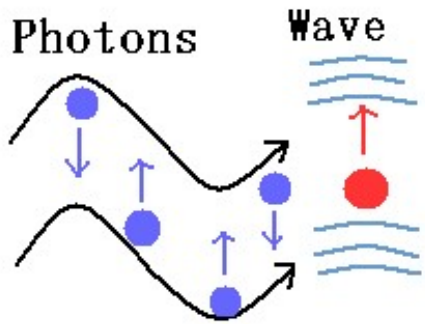

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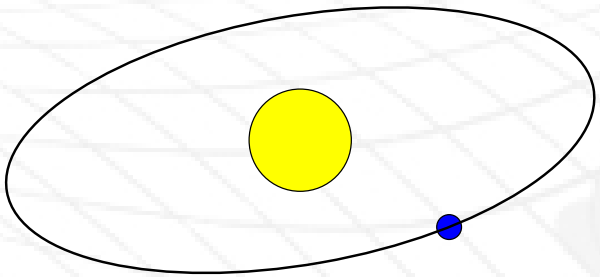
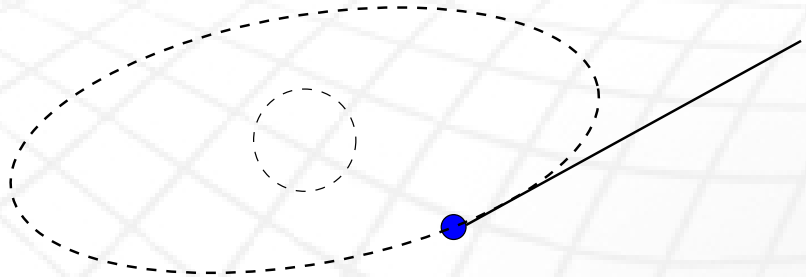
Let's suppose the sun disappears. How long does it take till we realize?





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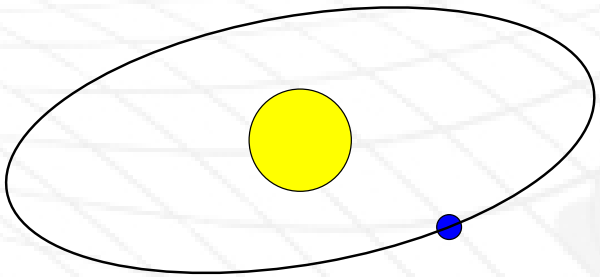

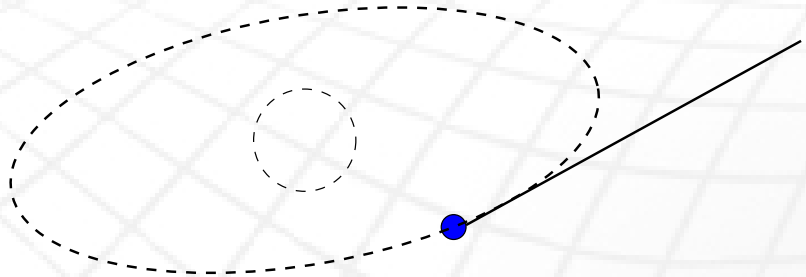

- It takes 8 minutes to the Sun light to reach Earth
- According to Newton (gravity is action at distance): Earth starts moving out tangentially 8 minutes BEFORE a terrestrial realizes the Sun light has switched off!
- Einstein: no physical signal can travel faster than c

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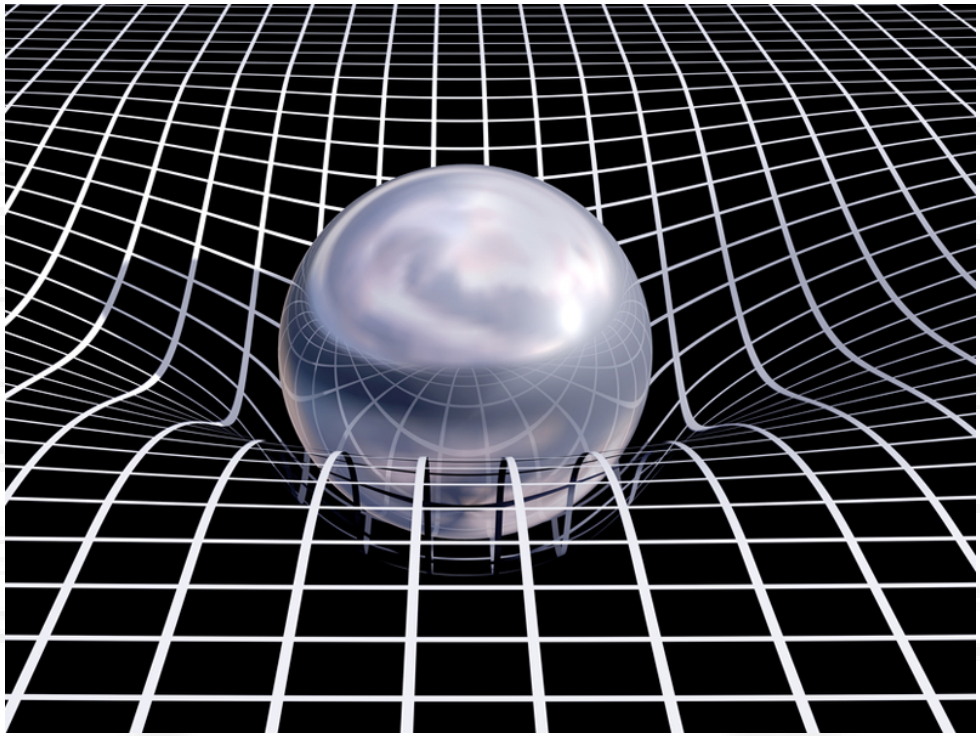

- The Space-time gets curved by the presence of "masses"

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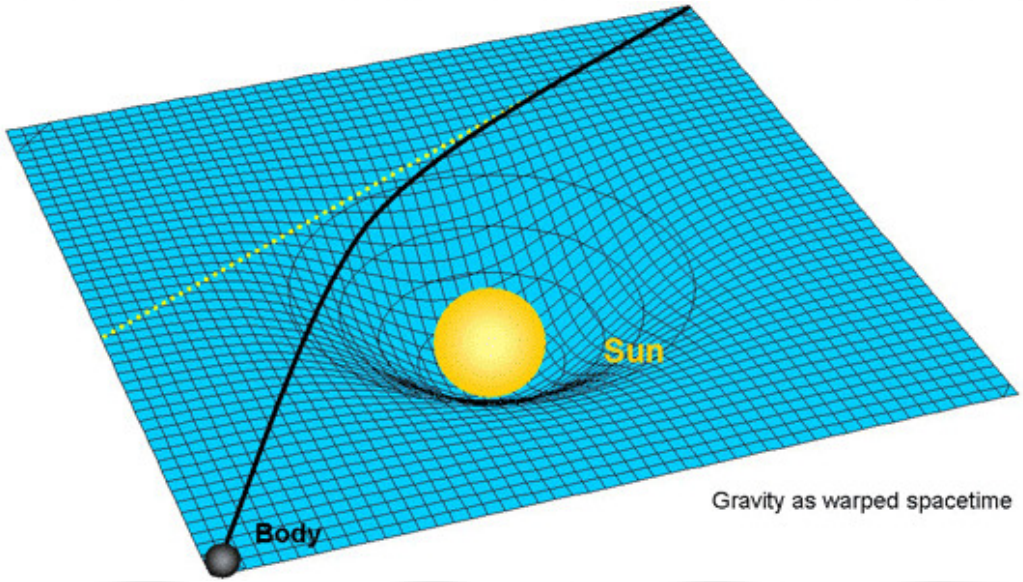

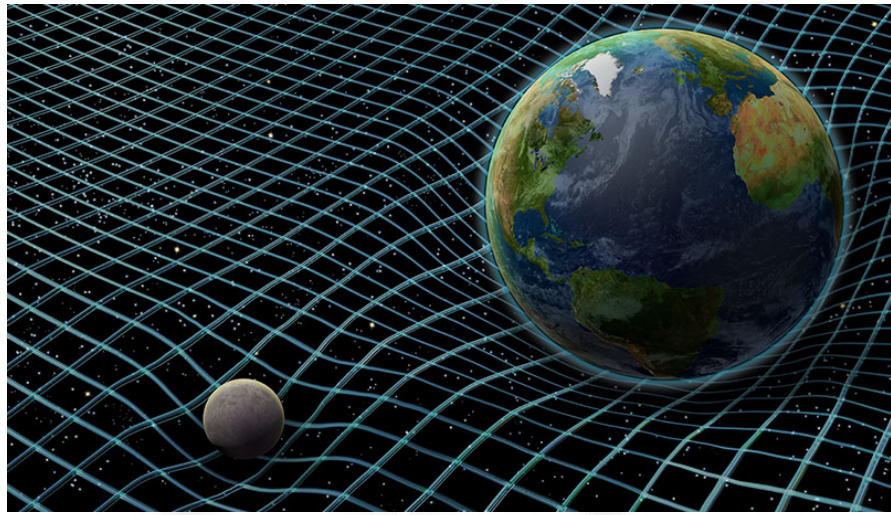

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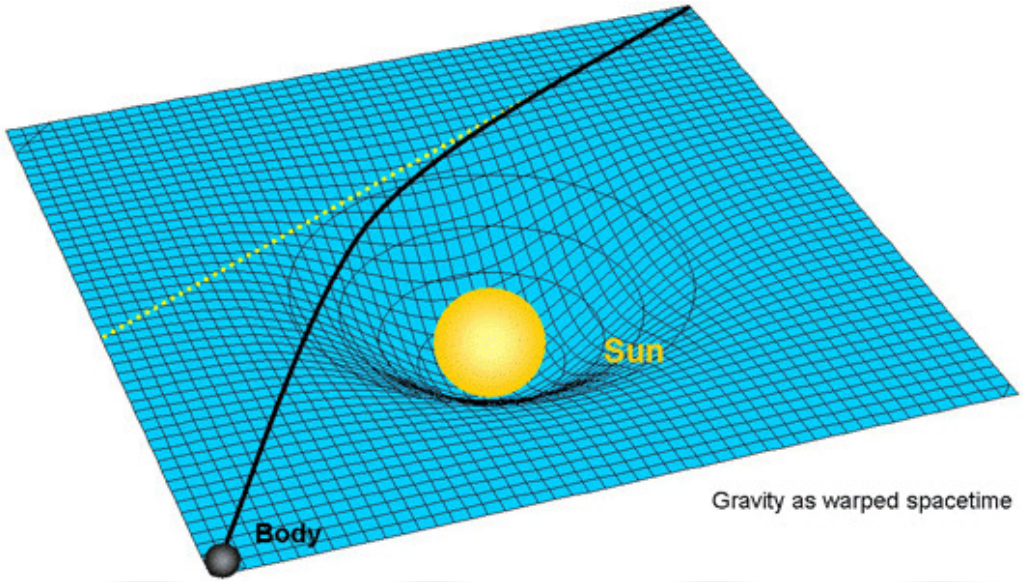

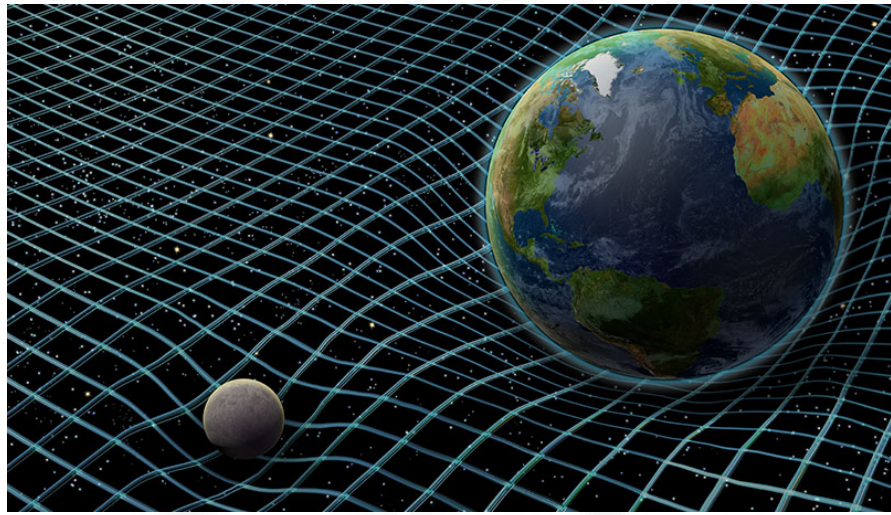

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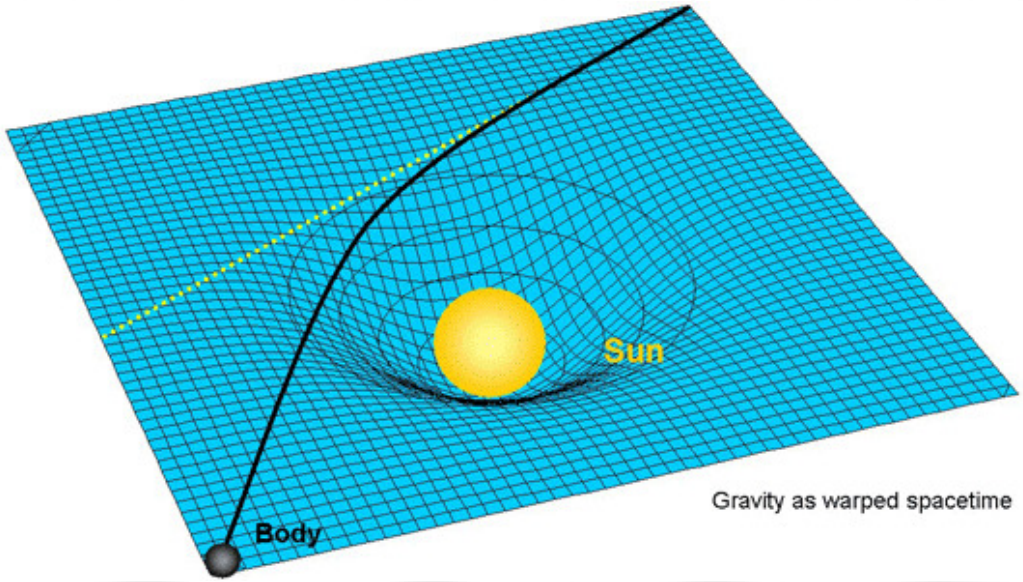

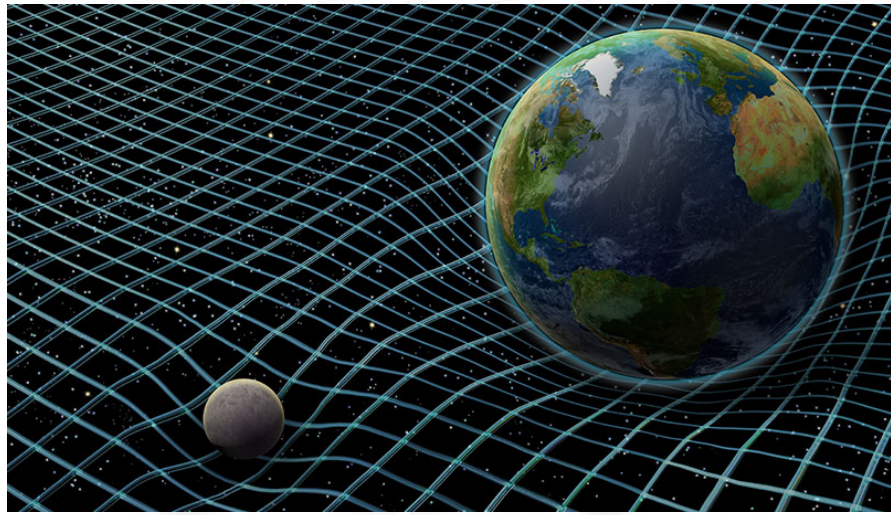

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- Orbits are lines described by test masses in curved space
- Test masses: masses that are small enough to give a negligible effect on the geometry

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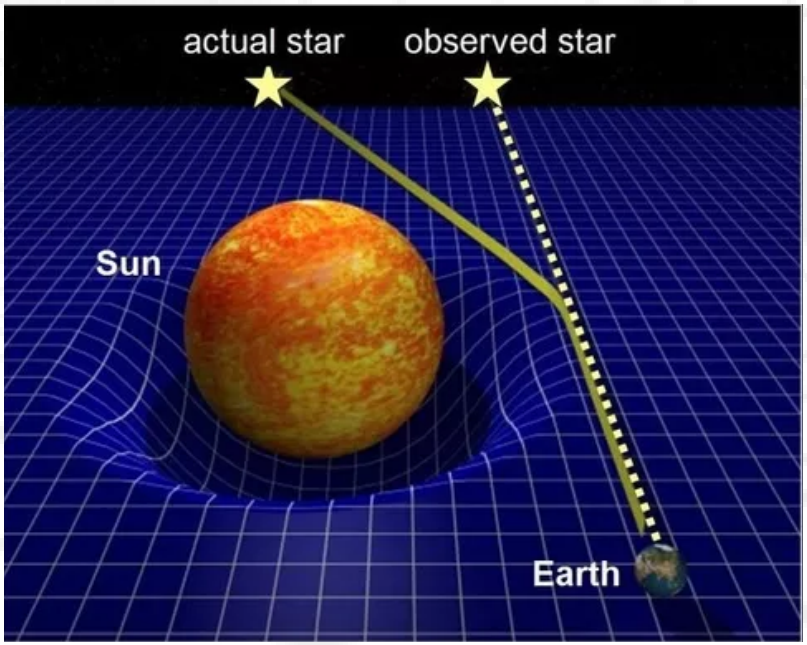

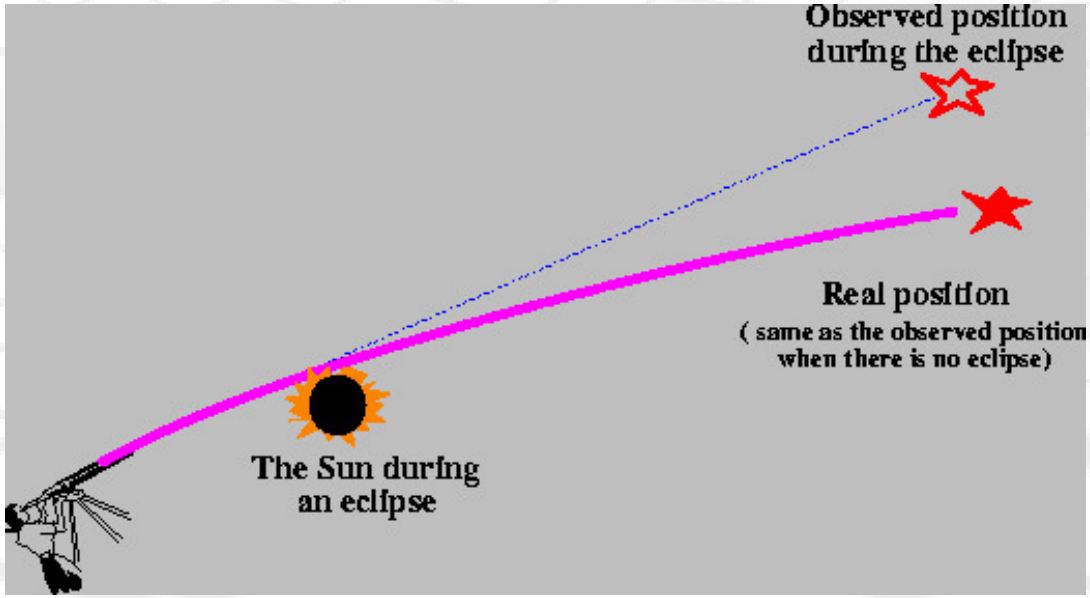

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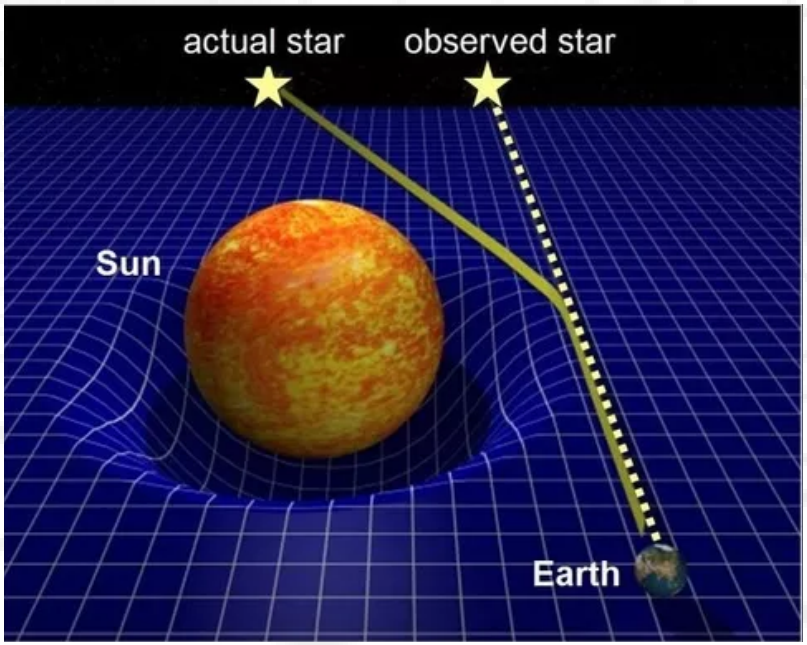

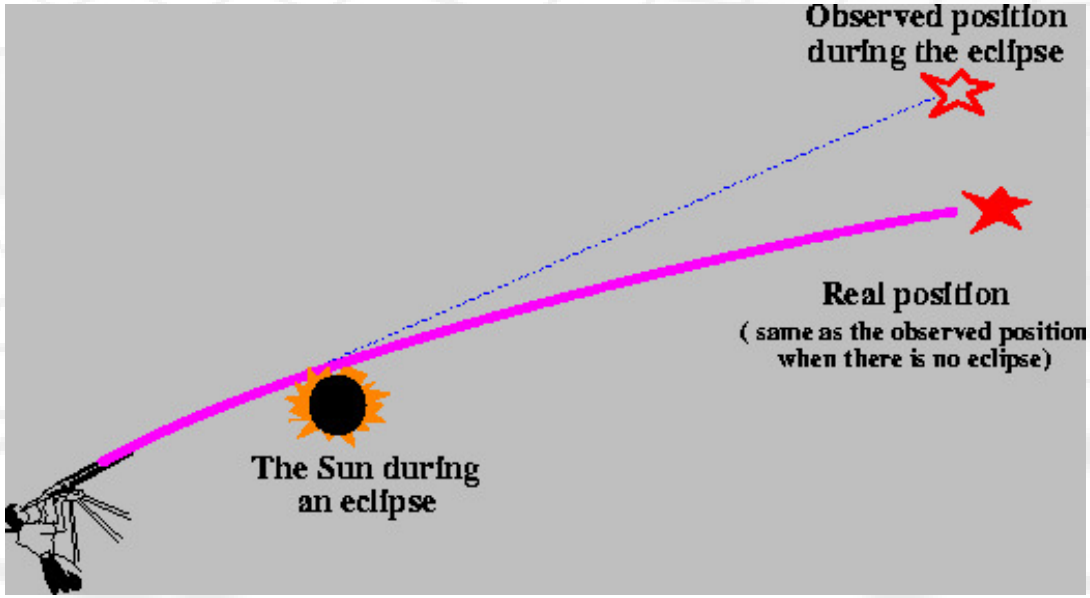

- In 1919 Eddington observed Light Bending during an eclipse

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A Field Theory

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- G = Newton's constant

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- Gravitational waves communicate the variation of geometry, due to the movement or variation of the source mass
- GW travel at the speed of light

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2 \right) h = 0$$

- Solution: waves whose fronts move with velocity c

What is General Relativity?

How stiff is the space-time?

- Analog: elastic force $F = -kx$

$$F \to T_{\mu\nu}, \quad x \to G_{\mu\nu} \implies k = \frac{c^4}{8\pi G} \approx 5 \times 10^{42} \ \frac{\mathrm{N}}{\mathrm{m}}$$

very stiff!!

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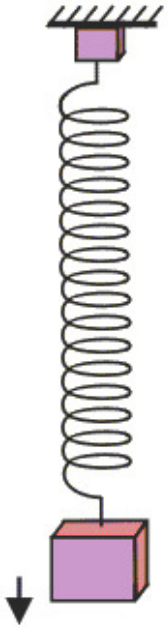

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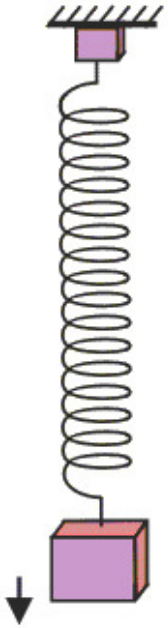

	M (**kg**)	x (**m**)
Milky Way	10^{42}	1
Sun	10^{30}	10^{-12}
Earth	10^{25}	10^{-17}

What is General Relativity?

A Field Theory

- Is there a gravitational analog of the photon?
- Einstein's equations are consistent with a "messenger" that is
  - massless: gravity, unlike other forces (e.g. nuclear force), has infinite range
  - spin two (boson)
- The graviton

Schwarzschild solution

First non-trivial solution of the Einstein's equations (1916)

- External geometry of a spherical boby (star, planet, black-hole)

$$ds^2 = -c^2 dt^2 \left(1 + \frac{2}{c^2}\phi(r)\right) + \frac{dr^2}{1 + \frac{2}{c^2}\phi(r)} + \cdots$$

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- small mass
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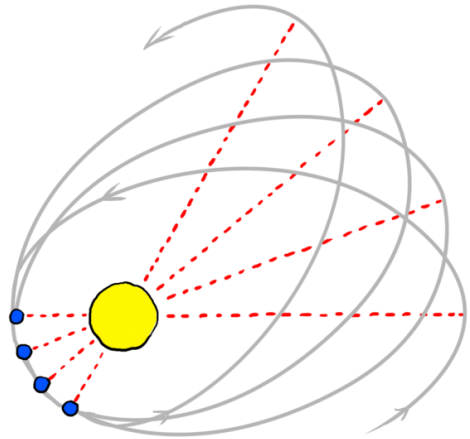
In the solar system, Mercury is the closest to Sun, yet $\frac{GM_\odot}{c^2 r} \sim 10^{-8}$.
Anomaly in the perihelion precession, NOT accounted for by classical corrections:

- Anomaly = 43 arcsec per century

  one part in 10 millions = $\frac{1}{10{,}000{,}000}$
- GR explains this anomaly



Schwarzschild solution

Light gets curved as well!

Schwarzschild solution

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- Radial velocity of a massless object (photon):

$$v_r^2 = 2\mathcal{E} - \frac{\ell^2}{r^2}\Big(1 \underbrace{+\frac{2}{c^2}\phi(r)}_{\text{relativistic correction}}\Big)$$

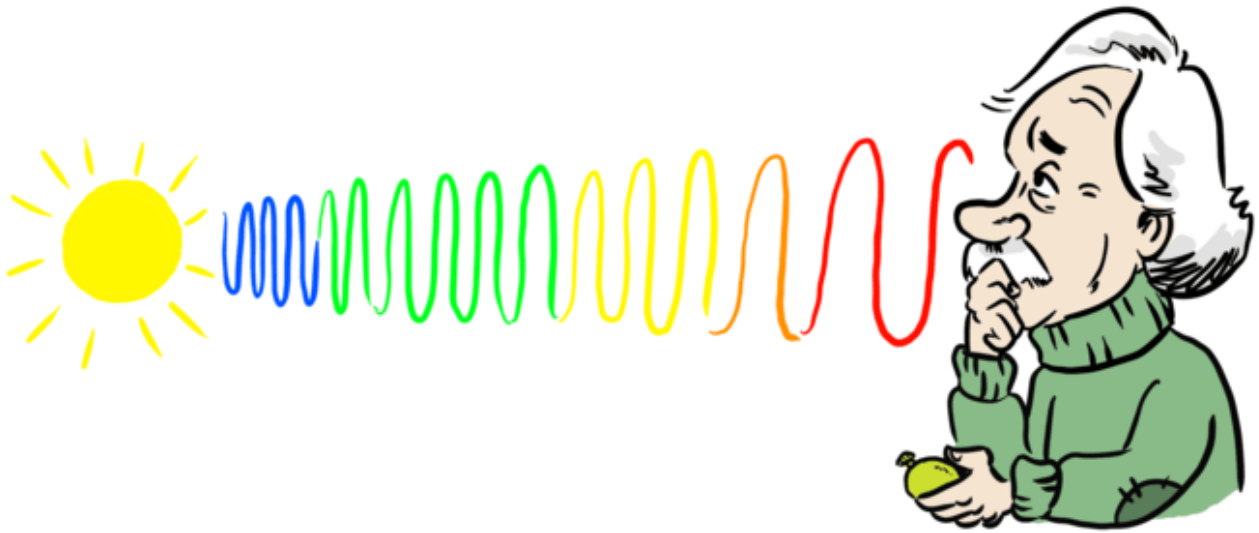
- When $\frac{GM}{c^2 r} << 1$: $v_r^2 = 2\mathcal{E} - \frac{\ell^2}{r^2}$

  trajectory is a straight line in polar coordinates
- In Newtonian gravity may try to predict light bending using the above massive expression (it is mass independent): it gives a wrong prediction of a factor 2

Einstein's GR predicts the correct value

Schwarzschild solution

Gravitational redshift: frequency gets smaller away from a massive source



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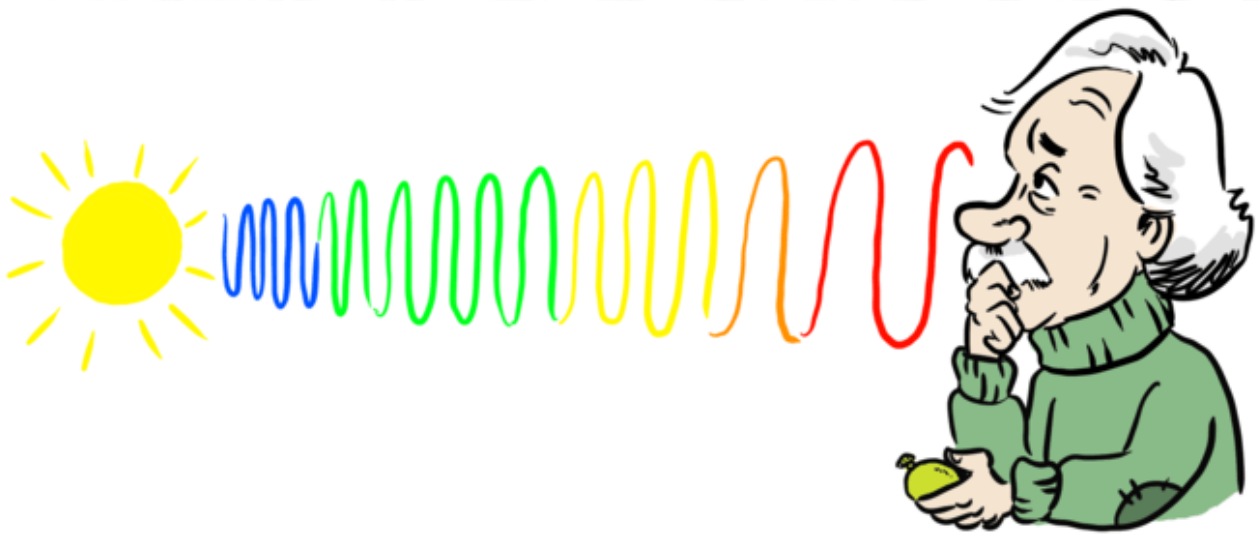

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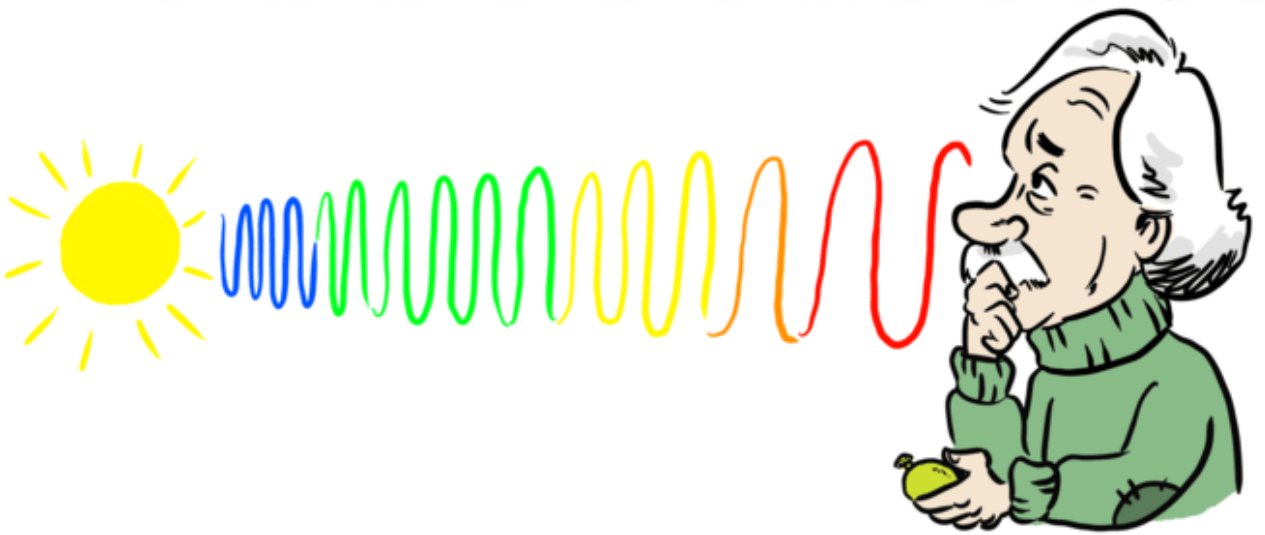

$$\omega < \omega_0 \,, \quad T > T_0$$

- the photon looses energy when climbing the gravitational potential: measured in Pound-Rebka experiment (1959)
- time passes slower in a stronger gravitational field: effect used in GPS

Cosmology

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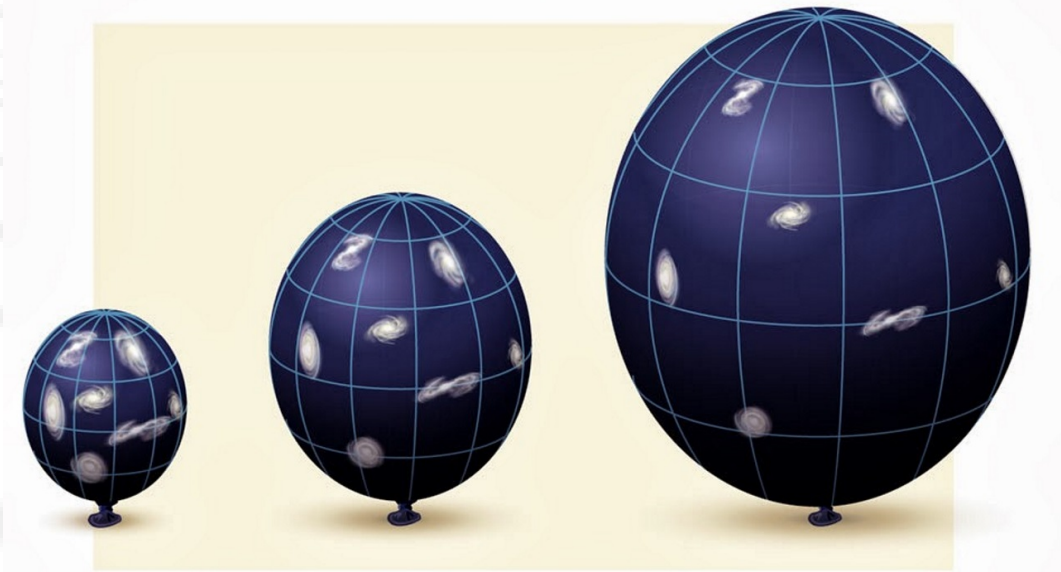

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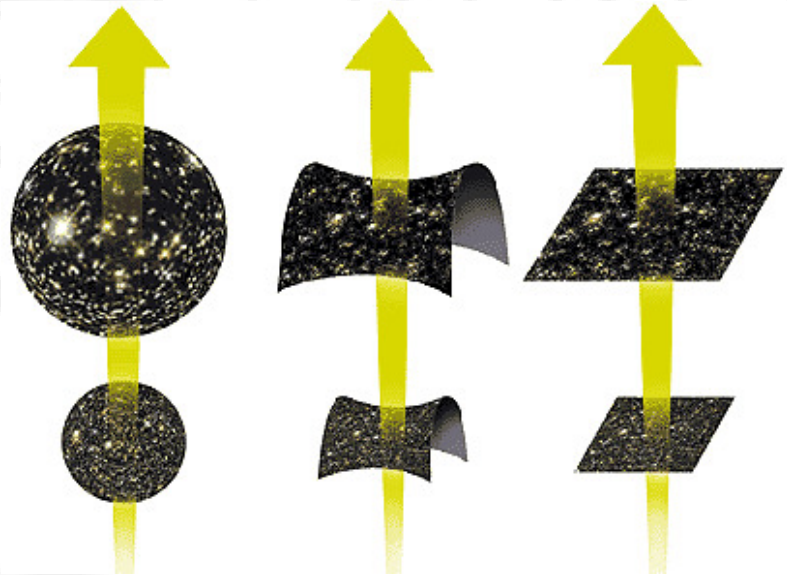

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The Nobel Prize in Physics 2011
Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

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Saul Perlmutter
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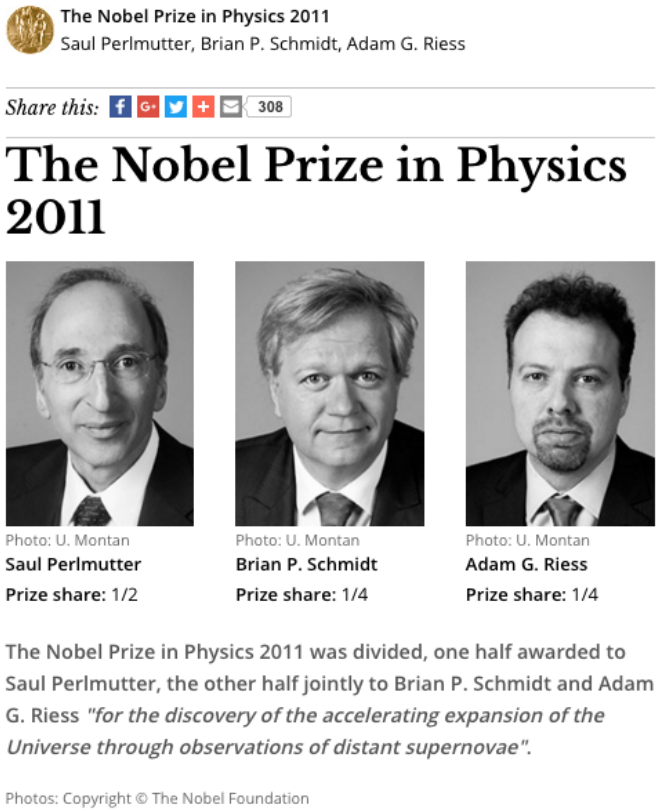

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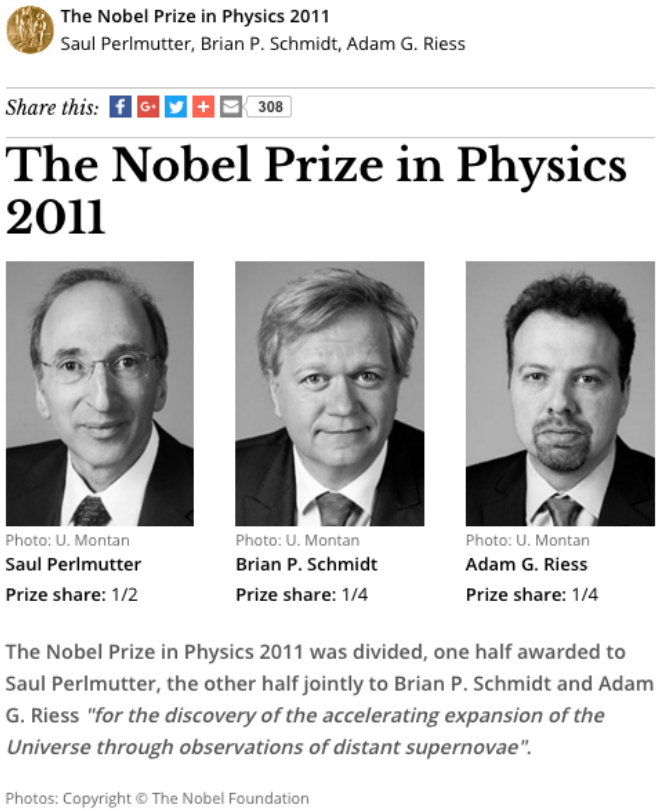
- A speculative alternative: massive graviton



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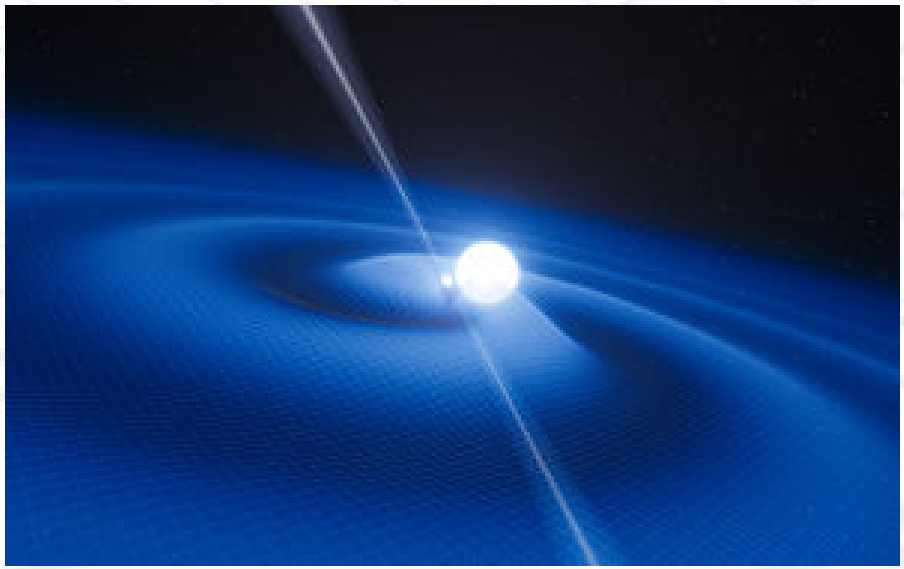
- Indirect test of existence of GW



The Nobel Prize in Physics 1993
Russell A. Hulse, Joseph H. Taylor Jr.

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The Nobel Prize in Physics 1993

Russell A. Hulse
Prize share: 1/2

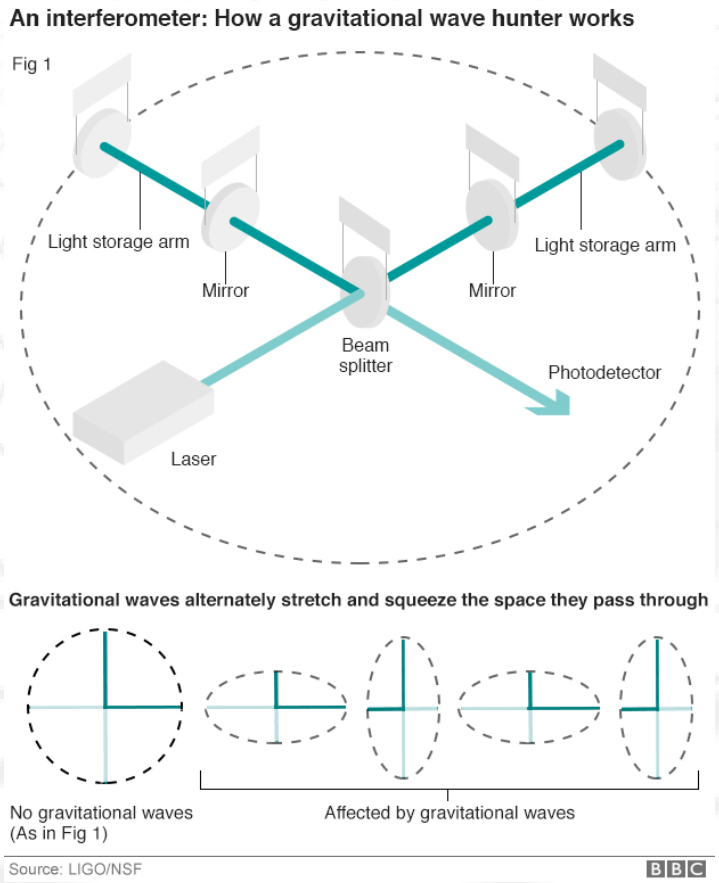
Joseph H. Taylor Jr.
Prize share: 1/2

The Nobel Prize in Physics 1993 was awarded jointly to Russell A. Hulse and Joseph H. Taylor Jr. *"for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"*

Gravitational Waves

2015: direct measurement of GW due to the merging of two back holes

$$\Delta L = hL\,, \quad h \sim 10^{-21}$$



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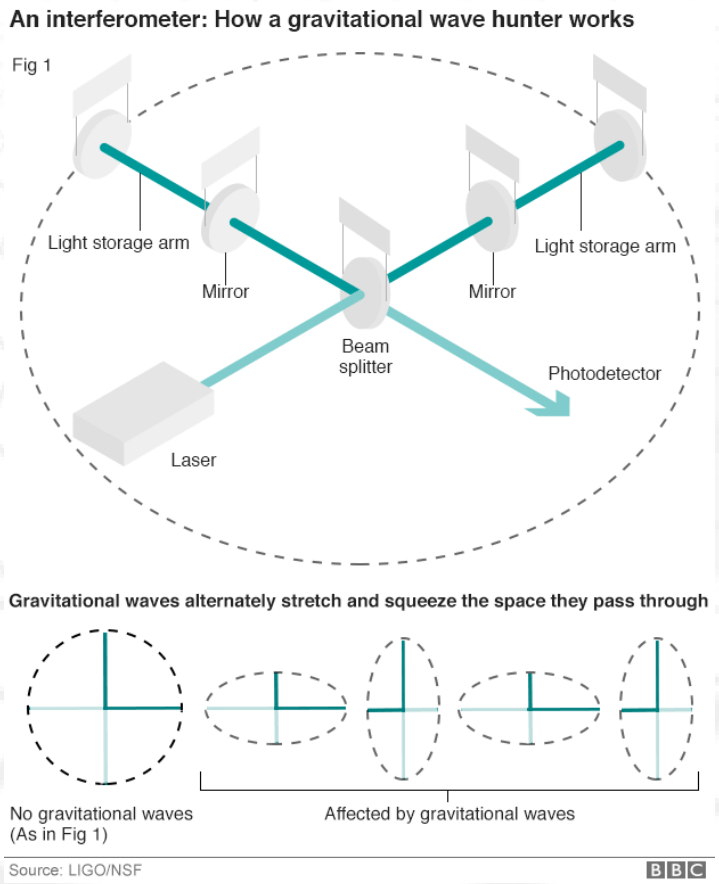

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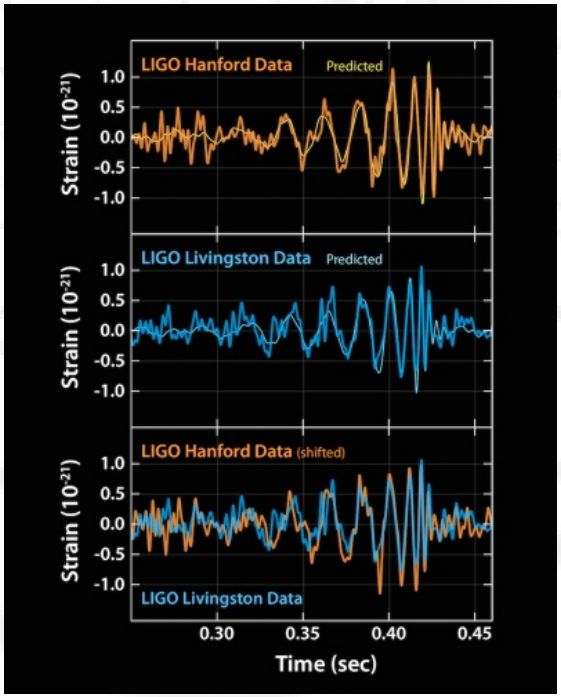
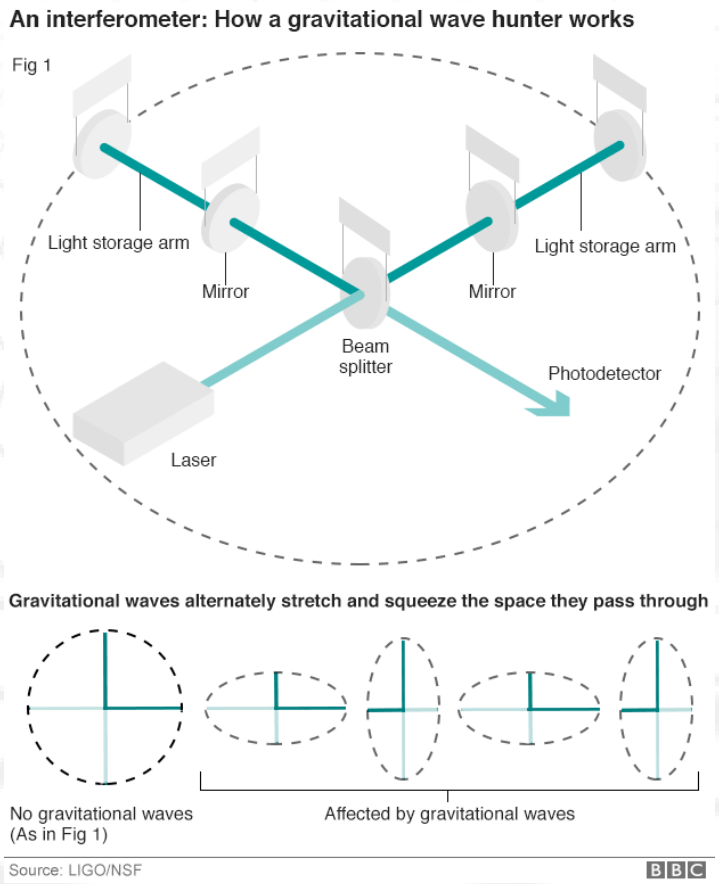
- $L_{phys} \sim 4$ km, $L_{eff} \sim 1000$ km





- $h_{min} \sim \dfrac{\lambda}{L_{eff}\sqrt{N}}$

Science 256 (1992) 325

Final Remarks

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  - Baryogenesis: how do we generate the observed amount of matter?
  - Future of GW detection: interferometer in space?
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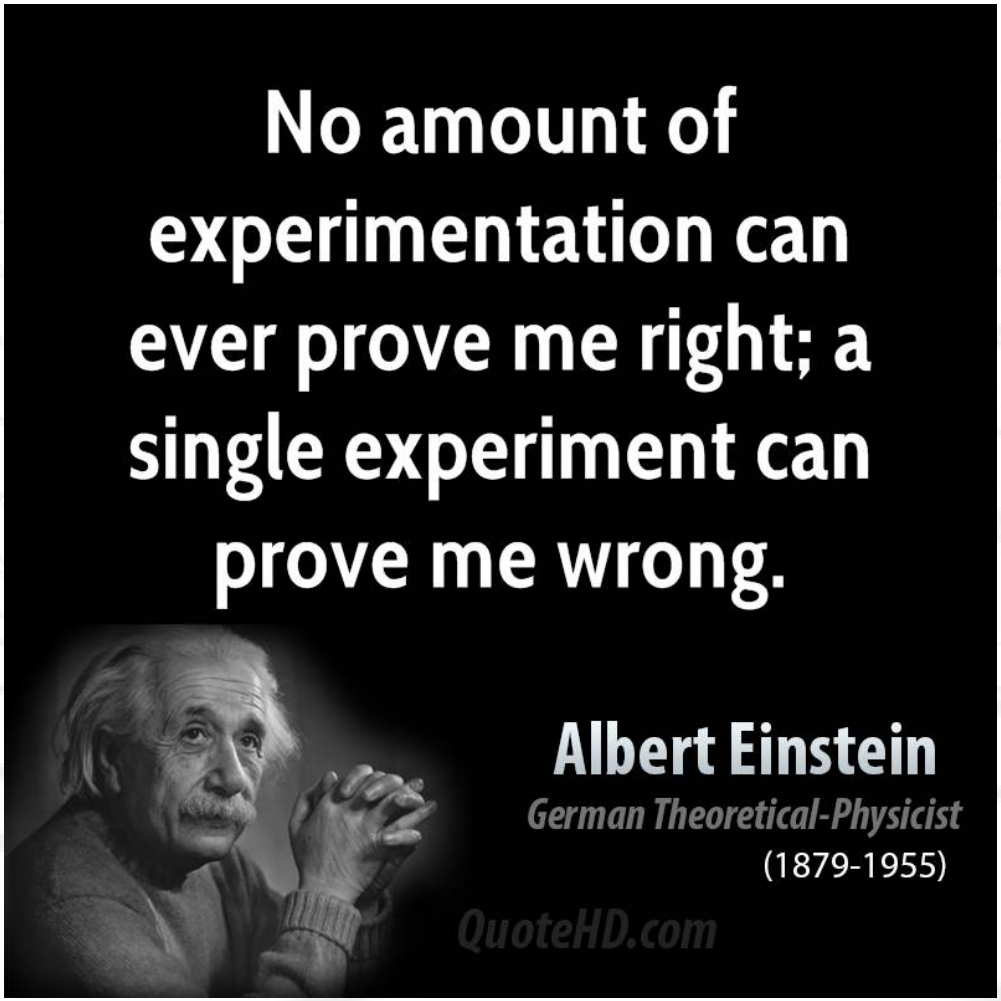
  - .....
- Quantum gravity is a long standing open problem

Thank you


No amount of experimentation can ever prove me right; a single experiment can prove me wrong.
Albert Einstein
German Theoretical-Physicist
(1879-1955)
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